Relations

A binary relation is one that takes 2 arguments. So, more formally, a binary relation on set some A is a subset of AxA (the cross product, or all possible pairs of elements of A). For example, if A = {1, 2, 3}, then the binary relation “=” (equality) on A is the following subset of pairs of elements of A: {<1, 1>, <2, 2>, <3, 3>}. For instance, the pair <1, 2> is not in the subset defined by “=”.

For all the definitions that follow, we assume the relations are binary.

A total relation on a set A is the case for which all a, b in A, either aRb or bRa. This means every pair of elements are comparable. The “less than”, or “<”, relation on the set of integers is total.

A reflexive relation on a set A is the case for which all a in A, aRa. For example, “equality”, or “=”, on the set of integers is reflexive. For example, 5 = 5.

An anti-reflexive, or irreflexive, relation on a set A is the case for which all a in A, then it is never the case that aRa. The strict “less than”, or “<”, relation on the set of integers is anti-reflexive. For example, 17 is not less than 17.

A symmetric relation on a set A is the case for which all a, b in A, then if aRb implies bRa. The “equality” relation on the set of integers is symmetric.

An anti-symmetric relation on a set A is the case for which all a, b in A, then aRb and bRa implies a=b. Note, an anti-symmetric relation is one where symmetry never holds. The strict “less than”, or “<”, relation on the set of integers is anti-symmetric. For example, since 4 < 5, then 5 is not < 4.

A transitive relation on a set A is the case for which all a, b, and c in A, then if aRb and bRc, then aRc. The strict “less than”, or “<”, relation on the set of integers is transitive. For example, since 42 < 69 and 69 < 1024, then 42 < 1024.

An anti-transitive relation is one which is never transitive. Let A = {rock, paper, scissors}. We define a relation where rock beats scissors, scissors beats paper, and paper beats rock. But, given rBs and sBp, it is not the case that rBp. It is easy to see that no other transitivity case is true either. This relation is also anti-reflexive and anti-symmetric.

An intransitive relation is one which is not always transitive. Let R be “distrusts” applied to countries. The US distrusts Iran, and Iran distrusts the UK, yet the US and the UK do not distrust each other. The US distrusts Iran, and Iran distrusts Iraq. Yet, the US distrusts Iraq also.

Order Relations

A partial order is reflexive, anti-symmetric, and transitive. This applies to the usual “less than or equal” relation on numbers. The strict “less than” relation is anti-reflexive.

A strict partial order is anti-reflexive.

A partial order that is total is a total order, linear order, or chain.

Conclusions

To specify an order relation, it seems we need to have 4 things:

1. The meaning of the ordering (e.g., less than, child of, instance of, etc.)
2. The set the ordering is applied to
3. Indicator of reflexivity (if no, the order is strict)
4. Indicator of totality (if yes, then the order is linear)

Items 3 and 4 above each need 2 values: yes or no.